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Development of Generalized Integral Index for Estimating Complex Impact of Major Factors of Winter Runoff Formation

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Abstract—Considered is a method of using the generalized integral index for the complex taking account of major factors of the winter runoff formation. An expert-statistical regression model is proposed and a method is worked out of optimizing the selection of its coefficients objectively taking account of the degree of the impact of each factor.

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When studying the hydrological processes and phenomena including the river runoff formation, different methods (balance, empirical, etc.) are used for taking account of major factors. One of the methods of taking into account can be the development and use of the generalized integral index indicating their complex impact [1].

Let us assume that there are k factors on the catchment territory under study influencing the runoff formation (for example, the relief, amount of precipitation, underlying surface, etc.). Then, any point of the catchment is characterized by the k-dimensional vector $\tilde{x} = (\tilde{x}^{(1)},\ldots, \tilde{x}^{(k)})$. Having measured the value of vector $\tilde{x}$ at N points, we get the matrix of initial data

$$B = \begin{pmatrix} x_1^{(1)} & \ldots & x_1^{(k)} \\ \ldots & \ldots & \ldots \\ x_N^{(1)} & \ldots & x_N^{(k)} \end{pmatrix}, \tag{1}$$

where x_i^j is the value of the jth separate index characterizing the ith factor.

It is difficult to interpret the multidimensional information contained in matrix B and to use it for analyzing the runoff formation conditions. In view of this, the generalized integral indices Z should be worked out. Certainly, these indices are the functions of k initial factors, i.e., $z = (x^{(1)},\ldots, x^{(k)})$. It is obvious that the main requirement for selecting Z is the optimum taking account of the contribution of each initial parameter.

The methods of creating the generalized integral index should be expert-statistical. In the statistical statement, this problem is classically solved using the methods of regression analysis and methods of principal components [1].

A method of expert-statistical regression model was used in the present paper for working out the generalized integral index. These models are used for analyzing the temporal and spatial series as well as for plotting the maps of statistical surfaces characterizing the spatial regularities of processes and phenomena. When working out the generalized integral index by the expert-statistical method, besides the initial data, the expert assessments $Z_{i,\,\mathrm{ex}}$, $i = 1, 2,\ldots, n$ should be available representing quantitative or ordinal values not measured directly and based on the judgments of specialists [2].

These data are used as a learning sample of the classic linear model of multiple regression of the following form:

$$Z_{i,\,\mathrm{ex}} = \theta_1 x_i'^{(1)} + \theta_2 x_i'^{(2)} + \ldots + \theta_k x_i'^{(k)} + \varepsilon_i, \tag{2}$$

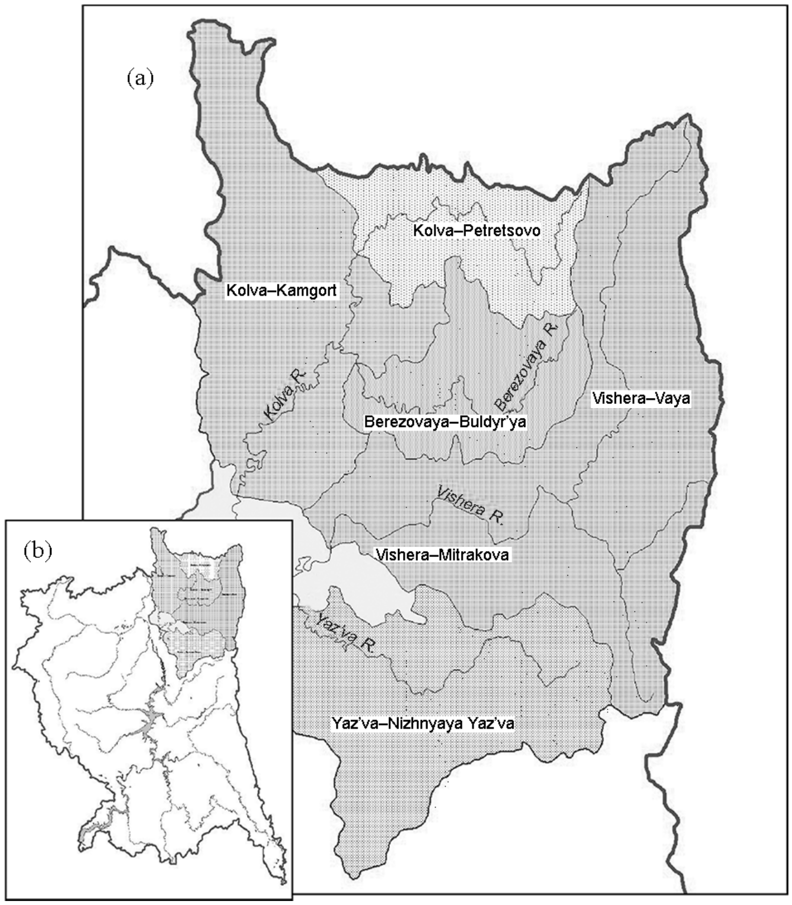

Fig. 1. (a) The Vishera River basin with indication of separate catchments and (b) its geographic location on the territory of the Votkinsk Reservoir.

where x_i^j is either normalized or rank parameters; ε_i are the random regression remainders meeting all the conditions of the classic model (i.e., they have the zero mean values and are mutually uncorrelated and homogeneous); $\theta = (\theta_1, ..., \theta_k)^{\mathrm{T}}$ is the vector of expert assessments, $\sum_{i=1}^{k} \theta_i = 1$.

Having assessed unknown coefficients $\theta_1, \theta_2, ..., \theta_k$ and having designated

$$z_i = \sum_{i=1}^{k} \theta_i x_i'^{(j)}, \tag{3}$$

the method of computing the values of generalized integral index can be determined.

Let us consider the possibility of using the generalized integral index indicating the complex impact of major factors on the winter runoff formation for the Vishera River catchment (Fig. 1) by means of plotting the expert-statistical regression model.

The precipitation fallen in autumn, the underlying surface features, the presence or absence of karst phenomena, and the relief type influence considerably the spatial distribution of the winter river runoff. To construct the regression model, all factors should be pairwise independent. Since the precipitation distribution depends on the catchment height, it is reasonable to estimate the relief effects using the slope angle. The most complete information about these factors can be obtained from digital topographic and thematic maps presented in the form of the raster model (GRID) and indicating the continuous variations of characteristics with the same size and number of cells (Fig. 2). Some essential features should be taken into account.

Firstly, each major factor affecting the winter runoff formation is described in different way, namely: the underlying surface is described qualitatively (heavy loamy, medium loamy, sandy loam soils, etc.), the

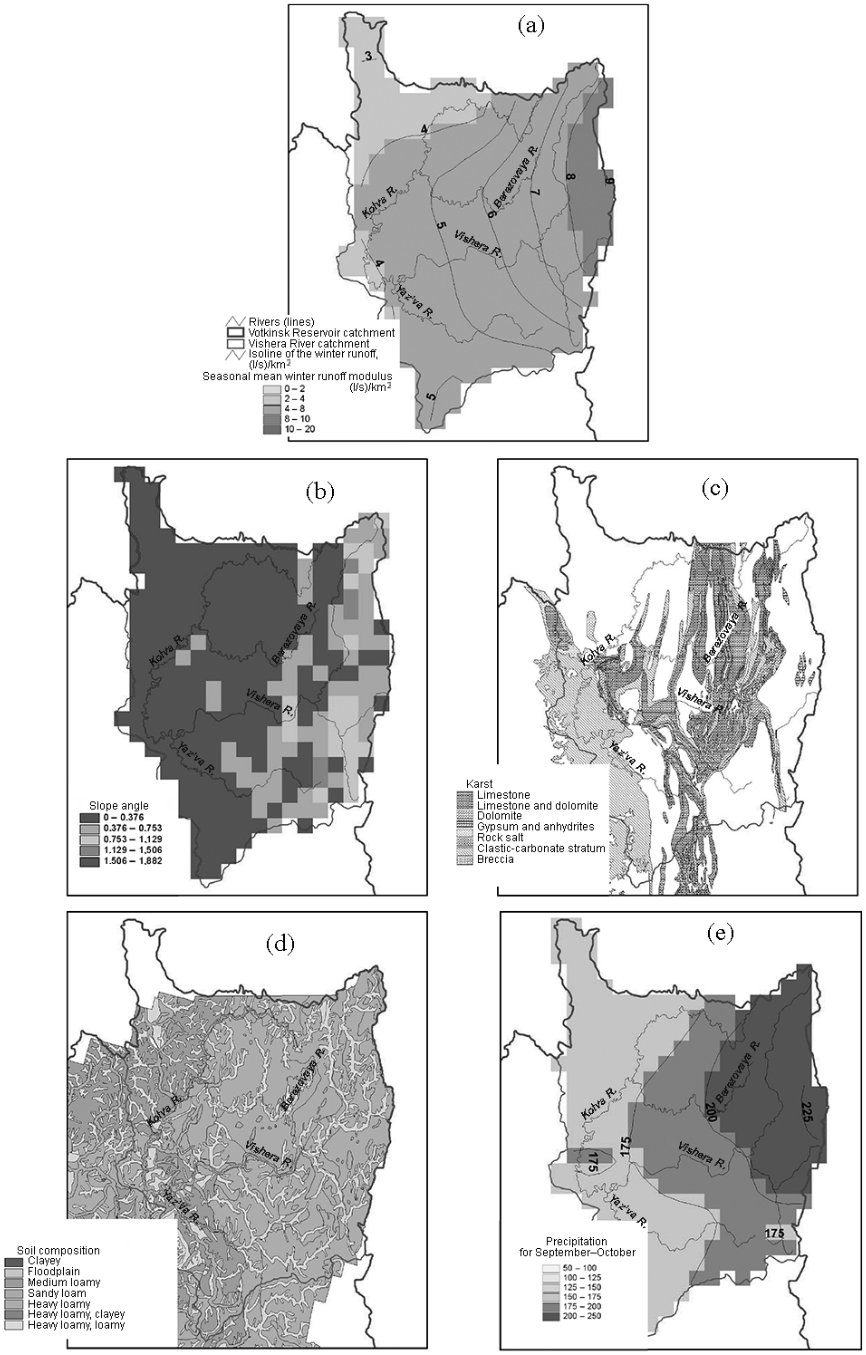

Fig. 2. Spatial distribution of (a) winter runoff and major factors influencing its formation: (b) slope angle, degree; (c) karst; (d) soil composition; (e) amount of precipitation for September–October, mm. The scale is 1:1 000 000.

slope angle and precipitation, quantitatively, and the presence or absence of the karst phenomena is noted. In this case, it is possible to use the ranking method enabling to combine the data into the single metrics (Table 1).

Secondly, when working out the integral index using empirical coefficients, the degree of the impact of separate factors should be taken into account. For this purpose, the method of expert assessments is used, where the priority is given either to separate factors or to their combinations (Table 2 and Fig. 3). In our opinion, the map of coefficients of correlation (r_i) between the spatial distribution of the winter runoff and the generalized integral index can be a criterion for assessing the appropriateness of using one or another

Table 1. Ranking of factors influencing the winter formation of the river runoff

Degree of impact	Rank	Factors of winter formation of the river runoff			
		karst phenomena	precipitation, mm	slope angle, degree	mechanical composition of soil
High	5		220–235	1–2	
Above medium	4		205–220	0.75–1.0	Floodplain
Medium	3		190–205	0.5–0.75	Sandy loam
Below medium	2		175–190	0.3–0.5	Medium loamy
Low	1	Yes	150–175	0–0.3	Heavy loamy
Absent	0	No			

Table 2. Variants of expert assessments of the degree of impact of separate factors

Factor	Equal degree of impact	Different degree of impact of factors			
		precipitation	slope angle	slope angle and precipitation	precipitation and mechanical composition of soil
Karst phenomena	0.25	0.1	0.1	0.025	0.05
Precipitation	0.25	0.6	0.3	0.45	0.5
Slope angle	0.25	0.2	0.5	0.5	0.15
Mechanical composition of soil	0.25	0.1	0.1	0.025	0.3

empirical variant [3]. The most acceptable method of creating the model map of isocorrelates between GRID-themes is a method associated with the formation of the sample within the basin under study [4], where each value of the raster corresponds spatially strictly to the value of another raster, i.e., pair arrays of values from the cells are actually formed. The computed correlation coefficients are placed to the centers of gravity of the catchments, and these data are interpolated for the whole territory under consideration.

The analysis of several different variants of the degree of the impact of separate factors on the winter runoff formation and of the maps of spatial distribution of coefficients of correlation between the runoff and the integral index, allowed to obtain the closest correlation (Fig. 4) in the case of using the following formula for computing the integral index:

$$Z_{i,\,\mathrm{ex}} = 0.1x_1' + 0.6x_2' + 0.2x_3' + 0.1x_4' + \varepsilon_i\,, \tag{4}$$

where x_1', x_2', x_3', and x_4' are the rank indices of karst phenomena, precipitation, the slope angle, and mechanical composition of soils, respectively, ε_i is regression remainders.

It was interesting to assess the possibility of optimizing the selection of coefficients taking account of the degree of the impact of separate factors on the winter runoff formation, i.e., of the coefficients corresponding to the closest dependence of the winter runoff on the integral index.

The optimization is based on the principle of using the criterion of maximization of the coefficient of correlation r_i between the variables y_i (runoff characteristics) and $Z_{i,\,\mathrm{ex}}$ (integral index). Thus, the objective function will be of the following form:

$$r_i = \frac{\sum_{j=1}^{n}(z_{ij} - \bar{z}_i)(y_{ij} - \bar{y}_i)}{\sqrt{\sum_{j=1}^{n}(z_{ij} - \bar{z}_i)^2(y_{ij} - \bar{y}_i)^2}} \to \max, \tag{5}$$

where i is the catchment; j is the cell belonging to the ith catchment; y_{ij} is the value of the river runoff in the jth cell; z_{ij} is the value of integral index in the jth cell; r_i is the coefficient of correlation between the values of integral index and runoff for the separate ith catchment.

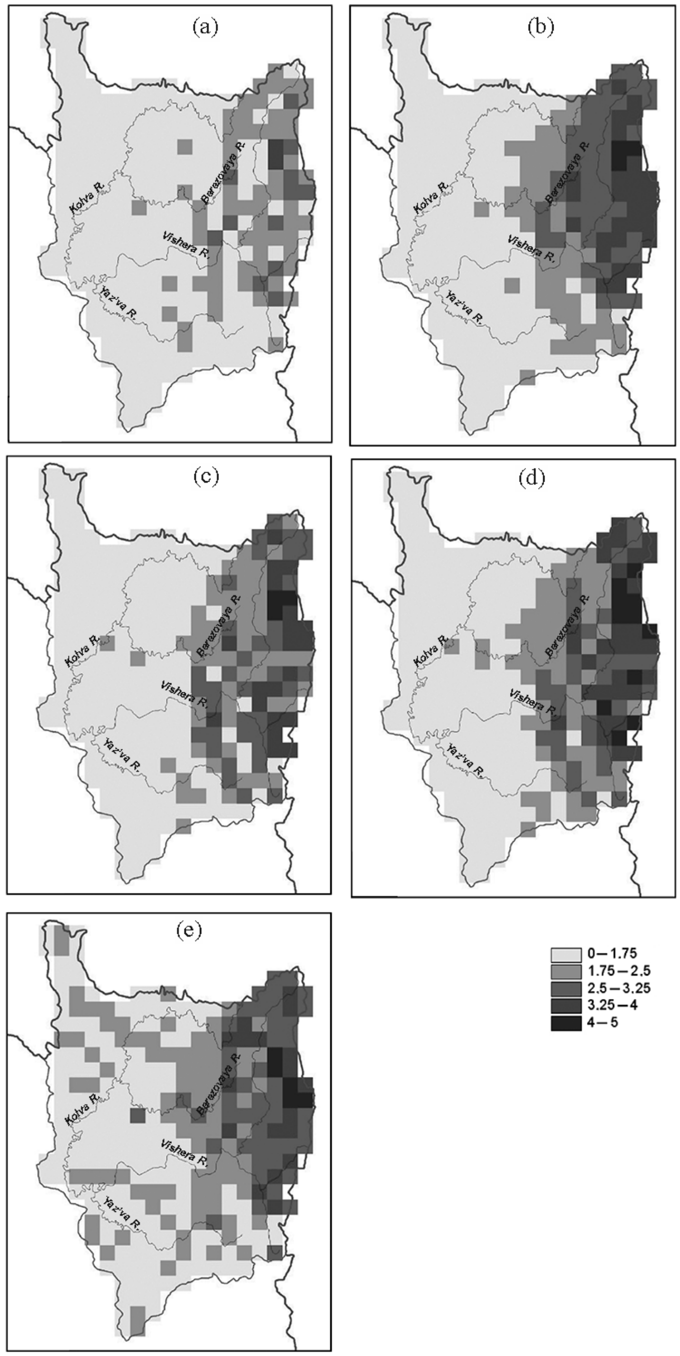

Fig. 3. Spatial distribution of generalized integral index variants. (a) Equal degree of impact; (b) prevalence of precipitation impact; (c) prevalence of the slope angle impact; (d) prevalence of the impact of precipitation and the slope angle; (e) prevalence of the impact of precipitation and mechanical composition of soil.

Let us accept the following limitations of the objective function: $r_i \le 1$; $\sum_{k=1}^{m} \theta_k = 1; 0 \le \theta_k \le 1$, where k is the factor; m is the number of factors.

To realize this purpose the authors used, the algorithms of automated search of the values of θ_k based on the nonlinear optimization (the Newton method and the conjugate gradient method) whose convergence depends, in particular, on the initial conditions (i.e., on how the sign gradations were presented before the optimization procedure [5]). The results of computations are presented in Table 3.

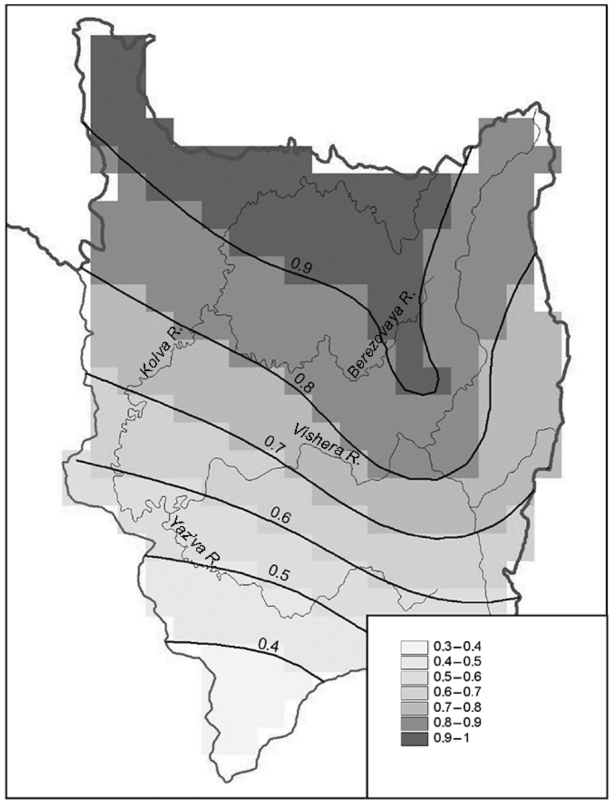

Fig. 4. Spatial distribution of coefficients of correlation between the runoff and the integral index.

Table 3. Values of correlation coefficients corresponding to the closest dependence of the winter runoff on the integral index

Factor, parameter	Values of coefficients taking account of the degree of impact of separate factors and of objective function for the catchments						
	Vishera–Mitrakova	Vishera–Vaya	Yaz'va–N. Yaz'va	Berezovaya–Buldyr'ya	Kolva–Petretsovo	Kolva–Kamgort	Vishera–Ryabinino
Karst phenomena	0.000	0.000	0.000	0.000	0.017	0.073	0.056
Precipitation	0.872	0.916	0.501	0.877	0.618	0.700	0.757
Slope angle	0.128	0.084	0.499	0.123	0.287	0.227	0.185
Mechanical composition of soil	0.000	0.000	0.000	0.000	0.077	0.000	0.001
Maximum values of r_i	0.933	0.865	0.529	0.918	0.949	0.903	0.929

Seven catchments, where the regular observations are carried out, are located on the territory of the Vishera River basin (Fig. 1). They differ in the size, location, and spatial inhomogeneity (Fig. 2) that is manifested in the different degree of the impact of some factors on the river runoff formation. The key factors for all the catchments are the precipitation and the slope angle while the effects of karst and mechanic composition of soils are practically not pronounced (Table 3). This agrees well with the above expert assessments.

Thus, the use of generalized integral index enables to carry out not only the complex taking account of major factors but also the estimation of the degree of the impact of each of them; the use of methods of optimization of coefficients of the correlation dependence enables to do it more objectively.

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